Warm up: Try These

Compound Interest: Present Value

1. Jason borrowed money that he will pay back in 7 years' time. The interest rate was 5.49%/a compounded monthly. He will repay $4050 after 7 years. How much money did Jason borrow?

$t = 7$
$r = 0.0549$
$A = 4050$
$c = 12$
$P = ?$

$n = t \times c$
$= 7 \times 12$
$= 84$
$i = \frac{0.0549}{12}$

$P = A(1+i)^{-n} = \frac{A}{(1+i)^n}$
$= 4050\left(1 + \frac{0.0549}{12}\right)^{-84}$
$\doteq 2760.17$

$\therefore$ Jason borrowed $2760.17.

2. A re-payment of $23 800 is due in 12 years for a loan. The interest rate is 6.5%/a, compounded quarterly. Determine the interest earned by the bank.

$I = ?$
$P = ?$
$A = 23\,800$
$r = 0.065$
$t = 12$
$c = 4$
$n = 48$
$i = \frac{0.065}{4}$

$P = A(1+i)^{-n}$
$= 23800\left(1 + \frac{0.065}{4}\right)^{-48}$
$\doteq 10\,978.68$

$\Rightarrow I = A - P$
$\doteq 23800 - 10\,978.68$
$\doteq 12\,821.32$

$\therefore$ The interest on the loan is about $12 821.32

7.4 Solving Financial Problems. Determining i and n.

Ex 1:
When Sean was born, his grandparents deposited $5000 in an account that pays interest compounded quarterly. No further deposits or withdrawals were made. On Sean's 25th birthday, the amount in the account was $22 160.23. What annual rate of interest did the account pay?

Sub #'s in straight away vs. Rearranging 1st; then sub

$A = 22\,160.23$
$P = 5000$
$t = 25$
$c = 4$
$n = 100$
$r = ?$
$i = ?$

$A = P(1+i)^n$
$22160.23 = 5000(1+i)^{100}$
$\frac{22160.23}{5000} = (1+i)^{100}$
$\sqrt[100]{4.432} \doteq 1+i$
$1.015 \doteq 1+i$
$i \doteq 0.015$

$A = P(1+i)^n$
$\frac{A}{P} = (1+i)^n$
$1+i = \sqrt[n]{\frac{A}{P}}$ or $\left(\frac{A}{P}\right)^{\frac{1}{n}}$
$i = \left(\frac{A}{P}\right)^{\frac{1}{n}} - 1$
$i = \left(\frac{22160.23}{5000}\right)^{\frac{1}{100}} - 1$
$i \doteq 0.015$

$\Rightarrow r = c \times i$
$\doteq 4 \times 0.015$
$\doteq 0.06$
$\doteq 6\%$

$\therefore$ The rate was about 6%

Ex 2:
Chad invests $1500 into a savings account that pays 6% compounded semi-annually. How long will it take Chad to triple his money?

$A = 3 \times 1500 = 4500$
$P = 1500$
$t = ?$
$c = 2$
$r = 0.06$
$i = 0.03$
$n = ?$

$A = P(1+i)^n$
$4500 = 1500(1.03)^n$
$\frac{4500}{1500} = 1.03^n$
$\log(3) = \log(1.03^n)$
$\log 3 = n \log 1.03$
$n = \frac{\log 3}{\log 1.03}$
$\doteq 37.17$

Recall:
n = # of yrs × # of compound pds/year
$n = t \cdot c$
$t = \frac{n}{c}$
$\doteq \frac{37.17}{2}$
$\doteq 18.6$
≈ 19

$\Rightarrow \therefore$ It will take about 18.6 years

Ex 3:

a) How long will it take for money to double at each interest rate compounded quarterly?

b) Is the time cut in half when the interest doubles?

i) 5%

$A = P(1+i)^n$

$\frac{A}{P} = 2$

$\frac{A}{P} = (1+i)^n$

$r = 0.05$

$c = 4$

$i = \frac{0.05}{4}$

$= 0.0125$

$2 = (1+0.0125)^n$

$2 = 1.0125^n$

$n = \frac{\log 2}{\log 1.0125}$

$\doteq 55.8$

$\Rightarrow t = \frac{n}{c}$

$\doteq \frac{55.8}{4}$

$\doteq 13.95$

13 yrs 0.95 years $\times \frac{12 \text{ months}}{\text{year}}$

11 months

$\therefore$ It would take about 13 years and 11 months to double your money.

ii) 10%

$\Rightarrow i = \frac{0.10}{4} = 0.025$

$2 = (1+0.025)^n$

$2 = 1.025^n$

$n = \frac{\log 2}{\log 1.025}$

$\doteq 28.07$

$t \doteq \frac{28.07}{4}$

$\doteq 7.02$

≈ 7 years

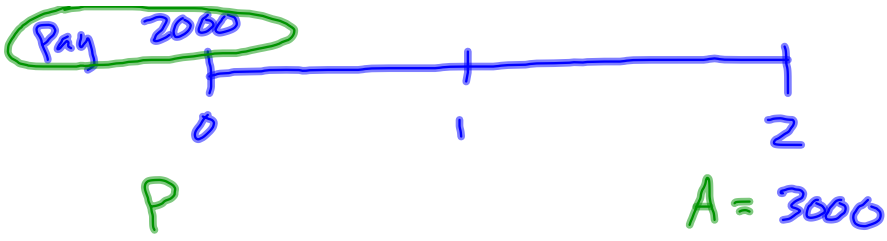
b) 13 years 11 months compared to 7 years

$\rightarrow$ NO, the time isn't halved when interest is doubled.


HMWK:
p 487
2, 3, 6 - 14

#7
p. 477

Tim



$P = ?$

$A = 3000$

$n = 4$

$i = \frac{0.105}{2}$

$$P = A(1+i)^{-n}$$

$$= 3000\left(1 + \frac{0.105}{2}\right)^{-4}$$

$$\doteq 2444.74$$

$\Rightarrow$ Total Payout $\doteq 2000 + 2444.74$

$\doteq 4444.74$

$\therefore$ Tim could pay $4444.74 today.

p. 477

#10

$P = A(1+i)^{-n}$

$t = 3$

$c = 12$

$n = 36$

$r = 5.25\%/a$

$i = \frac{0.0525}{12}$

$A = 3350$

$P = ?$

#12c
p.477

$P = 250\,000$

$i = \frac{0.105}{12}$

$n = 4 \times 12$

$= 48$

$A =$

$I = A - P$